Compound Interest

Finite Math

20 September 2018

Quiz

If some amount of money is deposited into a savings account with interest compounded biweekly, how many times is it compounded after 4 years?

Continuous Compound Interest

Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

$$\begin{aligned}
A &= P\left(1 + \frac{r}{m}\right)^{mt} \\
&= P\left(1 + \frac{r}{m}\right)^{mt \cdot \frac{r}{r}} \\
&= P\left(1 + \frac{r}{m}\right)^{\left(\frac{m}{r}\right)rt} \\
&= P\left(1 + \frac{1}{x}\right)^{xrt} \qquad \left(x = \frac{m}{r}\right) \\
&= P\left[\left(1 + \frac{1}{x}\right)^{x}\right]^{rt}
\end{aligned}$$

Continuous Compound Interest

$$A = P\left[\left(1 + \frac{1}{x}\right)^{x}\right]^{rt}$$

Now, if we let the number of compounding periods per year m get very very large, then x also gets very large, and we see that the future value becomes

$$A = Pe^{rt}.$$

Continuous Compound Interest

Definition (Continuous Compound Interest)

Principal P invested at an annual nominal rate r will have future value

$$A = Pe^{rt}$$

after time t (in years).

Compounding interest continuously gives the absolute largest amount of interest that can be accumulated in the time period t.

Continuous Compound Interest

Example

If $\$1,000$ *is invested at* 6% *interest compounded continuously, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Now You Try It!

Example

If $\$2,000$ *is invested at* 7% *compounded (a) annually, (b) quarterly, (c) monthly, (d) daily, (e) continuously, what is the amount after* 5 *years? Round answers to the nearest cent. (Assume* 365 *days in a year.)*

Solution

(a) $2805.10

(b) $2829.56

(c) $2835.25

(d) $2838.04

(e) $2838.14

Compound Interest

As before, we can use these compound interest models to figure out how much we should invest now to achieve a desired future value. We can also look to see how long something will take to mature given the principal, the growth rate, and the desired future value. The power rule for logarithms comes especially in handy here: $\log_b M^p = p \log_b M$.

Example

How long will it take $\$10,000$ *to grow to* $\$25,000$ *if it is invested at* 8% *compounded quarterly?*

Now You Try It!

Example

How long will it take money to triple if it is invested at (a) 5% *compounded daily? (b)* 6% *compounded continuously? (round to* 3 *decimal places)*

Solution

(a) $8,021$ *days (about* 21.975 *years)*

(b) 18.310 *years*

Compound Interest

We can also look to figure out the desired interest rate if we know the present value, the length of time, and the desired future value.

Example

The Russell Index tracks the average performance of various groups of stocks. On average, a $\$10,000$ *investment in mid-cap growth funds over a* 10*-year period would have grown to* $\$63,000$*. What annual nominal rate would produce the same growth if interest were compounded (a) annually, (b) continuously? Express answers as a percentage, rounded to three decimal places.*

Now You Try It!

Example

A promissory note will pay $\$50,000$ *at maturity* 6 *years from now. If you pay* $\$28,000$ *for the note now, what rate would you earn if interest were compounded (a) quarterly, (b) continuously?*

Solution

(a) 9.78%

(b) 9.66%